

FORWARD-BACKWARD CHARGE ASYMMETRY AT VERY HIGH ENERGIES

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Abstract

The impact of the electroweak radiative corrections on the value of the forward-backward asymmetry in e^+e^- annihilation into a quark-antiquark pair is considered in the double-logarithmic approximation at energies much higher than the masses of the weak bosons.

1 Introduction

The forward-backward asymmetry for the charged hadrons produced in e^+e^- annihilation at high energies in vicinity of the Z -boson mass has been the object of intensive theoretical and experimental investigation. It is interesting to make a theoretical analysis of this phenomenon for much higher energies where the leading, double-logarithmic (DL) contributions come not only from integrating over virtual photon momenta but also from virtual W and Z bosons. Indeed, at the annihilation energies much higher than 100 GeV, when masses of virtual electroweak bosons are small compared to their momenta, the initial $SU(2) \times U(1)$ symmetry is restored in a certain sense and accounting for higher loop DL contributions involving the W, Z bosons is no less important than the standard accounting for the photon DL contributions. The value of the asymmetry at such energies would be expressed rather through the Casimir operators of the electroweak gauge group than through electric charges. In a sense, studying the forward-backward asymmetry at such energies is one of the simplest ways to see the total effects of contributions of the electroweak radiative corrections of higher orders. Our theoretical study can be useful in future when Next linear colliders will explore e^+e^- annihilation at very high energies, probing further the Standard Model and eventually looking for New Physics.

In accordance with the present theoretical conceptions, we divide investigation of the annihilation into two stages: first we calculate the sub-process: e^+e^- -annihilation into a quark-antiquark pair (which is studied with perturbative methods) and then, numerically, account for hadronization effects which are quite different for converting the quarks into mesons and baryons. We consider e^+e^- annihilate into two hadronic jets in the kinematics when the leading particles of every jet goes in cmf close to the beam axis, so they are within the cones with opening angles $\theta \ll 1$ and the axes around the e^- and e^+ directions. We obtain that the forward-backward asymmetry manifests itself as follows: the number of the hadrons with the positive electric charges, N_+ in the cone around the e^+ -direction exceeds the number of the negatively charged hadrons, N_- (see Ref. [1]). It is depicted in Fig. 1. The opposite effect is true for the other cone, around the e^- direction. The space outside of the cones is neutral. The numerical evaluations of the asymmetry are plotted in Fig. 2 separately for the produced mesons and baryons. In particular, Fig.2 shows that the value of the asymmetry is high enough to be measured at energies ≈ 1 TeV and grows steeply with energy.

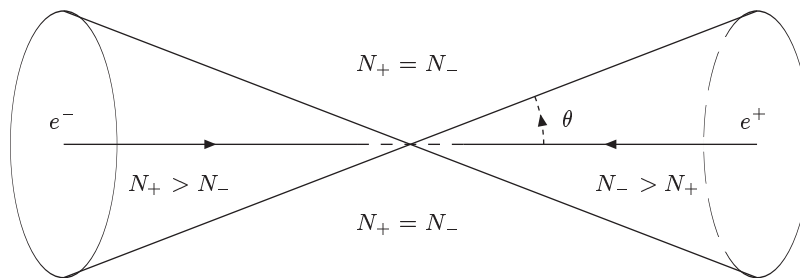


Figure 1: *Relations between charged hadrons in different angular regions.*

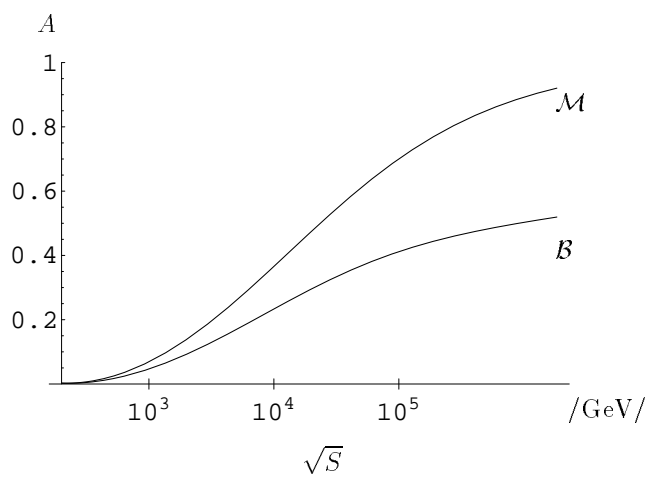


Figure 2: *Estimation of charge asymmetry A of leading charged hadrons in e^+e^- annihilation: The curve \mathcal{M} is for the meson asymmetry and the curve \mathcal{B} is for the asymmetry of barions.*

In order to arrive at this result, let us consider first the basic sub-process of the annihilation: $e^+(p_2)e^-(p_1)$ annihilation into a quark $q(p_3)$ and its antiparticle \bar{q} . All these particles can belong either to the $SU(2)$ doublets or to the singlets. Our first goal is to calculate the scattering amplitudes of the annihilation in the t - kinematics where

$$s = (p_1 + p_2)^2 \gg t = (p_3 - p_1)^2 \quad (1)$$

and in the u - kinematics:

$$s = (p_1 + p_2)^2 \gg u = (p_4 - p_1)^2 . \quad (2)$$

In order to account for DL contributions to all orders in the electroweak couplings, we use the evolution equations with respect to the infrared cut-off. This cut-off M is chosen in the transverse momentum space so that momenta of all virtual particles obey

$$k_{i\perp} > M . \quad (3)$$

Although only the Feynman graphs with virtual photons can have the infrared divergencies, it is convenient to keep the restriction (3) for momenta of all virtual particles, assuming that

$$M \geq M_Z \approx M_W . \quad (4)$$

With assumptions of Eqs. (34), one can neglect all masses and be safe of the infrared singularities at the same moment. On the other hand, the scattering amplitudes now depend on M . It makes possible to evolve them in M and to put $M = M_Z \approx M_W$ in the final expressions. As DL contributions appear in the regions where $k_{i\perp}$ obey the strong inequalities of the kind $k_{i\perp} \gg k_{j\perp}$, it is always possible to find the virtual particle with minimal ($\equiv k_\perp$) transverse momentum in every such a region. Obviously, only integration over k_\perp involves M as the lowest limit. Integrations over other transverse momenta are M -independent. DL contributions of the softest particles can be factorized. It allows to compose infrared evolution equations (IREE) for the scattering amplitudes. The most difficult is the case when both the initial electron and the final quark belong to the $SU(2)$ doublets. In order to simplify the IREE, one can use the $SU(2)$ symmetry restored at such high energies and consider annihilation of lepton-antilepton pair into a quark-antiquark pair. After that, it is convenient to expand the scattering amplitude into the sum of the irreducible $SU(2)$ representations, using the standard projection operators multiplied by the invariant amplitudes A_j , ($j = 1, 2, 3, 4$). At last, in order to calculate the invariant amplitudes in kinematics (1, 2), it is convenient to use the Mellin transform

$$A_j = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} \left(\frac{s}{\kappa} \right)^\omega F_j(\omega) \quad (5)$$

where $\kappa = t, j = 1, 2$ for A_j in kinematics (1) and $\kappa = u, j = 3, 4$ when the kinematics is the u -inematics of Eq. (1). In the case of the collinear kinematics where $\kappa = M^2$, amplitudes F_j obey (we consider amplitudes with the positive signatures only):

$$\omega F_j(\omega) = a_j + \frac{b_j}{8\pi^2} \frac{dF_j^{(+)}(\omega)}{d\omega} + \frac{c_j}{8\pi^2} \left[F_j^{(+)}(\omega) \right]^2 , \quad (6)$$

with a_j, b_j, c_j being numerical factors:

$$\begin{aligned} a_1 &= \frac{3g^2 + g'^2 Y_1 Y_2}{4}, & a_2 &= \frac{-g^2 + g'^2 Y_1 Y_2}{4}, \\ a_3 &= \frac{-3g^2 + g'^2 Y_1 Y_2}{4}, & a_4 &= \frac{g^2 + g'^2 Y_1 Y_2}{4}, \end{aligned} \quad (7)$$

$$\begin{aligned} b_1 &= \frac{g'^2 (Y_1 - Y_2)^2}{4}, & b_2 &= \frac{8g^2 + g'^2 (Y_1 - Y_2)^2}{4}, \\ b_3 &= \frac{g'^2 (Y_1 - Y_2)^2}{4}, & b_4 &= \frac{8g^2 + g'^2 (Y_1 + Y_2)^2}{4}, \end{aligned} \quad (8)$$

$$c_1 = c_2 = 1, \quad c_3 = c_4 = -1. \quad (9)$$

Solutions to Eq. (6) can be expressed in terms of the Parabolic cylinder functions D_p :

$$F_j(\omega) = \frac{a_j}{\lambda_j} \frac{D_{p_j-1}(\omega/\lambda_j)}{D_{p_j}(\omega/\lambda_j)} \quad (10)$$

where $p_j = a_j c_j / b_j$ and $\lambda_j = \sqrt{b_j / (8\pi^2)}$.

When the scattering angles are becoming larger so that

$$s \gg -\kappa \gg \mu^2, \quad (11)$$

the invariant amplitudes A_j are expressed in terms of F_j in the following way:

$$A_j(\rho, \kappa) = a_j S_j \int_{-\infty}^{\infty} \frac{dl}{2\pi i} e^{\lambda_j l(\rho - \eta)} \frac{D_{p_j-1}(l + \lambda_j \eta)}{D_{p_j}(l + \lambda_j \eta)} \quad (12)$$

where $\eta = \ln(\kappa/\mu^2)$ and factors h_j can be taken from Ref. [1]:

$$\begin{aligned} h_1 &= \frac{3g^2 + g'^2 Y_l Y_q}{2}, & h_2 &= \frac{-g^2 + g'^2 Y_l Y_q}{2}, \\ h_3 &= \frac{3g^2 - g'^2 Y_l Y_q}{2}, & h_4 &= \frac{-g^2 - g'^2 Y_l Y_q}{2}. \end{aligned} \quad (13)$$

The factor S_j in Eq. (12) is the Sudakov form factor. Actually it is a product of the Sudakov form factors of the leptons and of the quarks. As follows from Eqs. (8,13), Due to the gauge invariance, it does not depend on j , i.e. is same for all invariant amplitudes. It can be seen also explicitly from Eqs. (8,13),

$$S = \exp \left[-\frac{1}{8\pi^2} \left(\frac{3}{2} g^2 + \frac{Y_l^2 + Y_q^2}{4} g'^2 \right) \frac{\eta'^2}{2} \right]. \quad (14)$$

This form factor accumulates DL contributions of soft virtual EW bosons and vanishes in the final expressions for the cross sections when bremsstrahlung of soft EW bosons with the cmf energies ϵ_k , $\mu < \epsilon_k < \sqrt{\kappa}$, is taken into account.

According to the results of Ref. [1], the amplitude for the forward $e^+e^- \rightarrow u\bar{u}$ -annihilation, M_u^F is expressed in terms of amplitudes A_3, A_4 of Eq. (12):

$$M_u^F = \frac{A_3 + A_4}{2}, \quad (15)$$

whereas the backward amplitude M_u^B for the same quarks is equal to the amplitude A_4 of Eq. (12):

$$M_u^B = A_4. \quad (16)$$

Similarly, the forward amplitude M_d^F for $e^+e^- \rightarrow d\bar{d}$ is

$$M_d^F = \frac{A_1 + A_2}{2}, \quad (17)$$

while the backward amplitude for this process is

$$M_d^B = A_2. \quad (18)$$

We use the following terminology: by the forward kinematics for $e^+e^- \rightarrow q\bar{q}$ -annihilation we mean that quarks with positive electric charges, u and \bar{d} , are produced around the initial e^+ - beam, in the cmf, within a cone with a small opening angle θ ,

$$1 \gg \theta \geq \theta_0 = \frac{2M}{\sqrt{s}} \quad (19)$$

By backward kinematics we means just the opposite – the electric charge scatters backwards in a cone with the same opening angles.

The differential cross section $d\sigma_F$ for the forward annihilation is

$$d\sigma_F = d\sigma^{(0)} [|M_u^F|^2 + |M_d^F|^2] \equiv d\sigma^{(0)} [F_u + F_d] , \quad (20)$$

and similarly, the differential cross section $d\sigma_B$ for the backward annihilation is

$$d\sigma_B = d\sigma^{(0)} [|M_u^B|^2 + |M_d^B|^2] \equiv d\sigma^{(0)} [B_u + B_d] , \quad (21)$$

where $d\sigma^{(0)}$ stands for the Born cross section, though without couplings. We define the forward-backward asymmetry as:

$$A \equiv \frac{d\sigma_F - d\sigma_B}{d\sigma_F + d\sigma_B} = \frac{F - B}{F + B} \quad (22)$$

where

$$F = F_u + F_d, \quad B = B_u + B_d . \quad (23)$$

Contributions to the asymmetry from right leptons and quarks can be easily obtained in a similar way. Then, using the standard programmes for hadronisation[3] and doing numerical calculations, we obtain the forward-backward asymmetry for e^+e^- annihilation into charged mesons and baryons. The results are plotted in Fig. 2.

Finally, we would like to note that when the annihilation energy is high enough to produce, in addition to the two quark jets, W or Z bosons with energies $\gg 100$ GeV, it turns out that the cross section $\sigma^{(n\gamma)}$ of n hard photon production, the cross section $\sigma^{(nZ)}$ of nZ boson production and the cross section $\sigma^{(nW)}$ of nW boson production obey very simple asymptotical relations (see Ref. [2]):

$$\frac{\sigma^{(nZ)}}{\sigma^{(n\gamma)}} \approx \tan^{2n} \theta_W , \quad (24)$$

$$\frac{\sigma^{(n\gamma)}}{\sigma^{(nW)}} \sim s^{-0.36} . \quad (25)$$

The ratios of these cross sections as functions of the annihilation energy are plotted in Figs. 3,4.

2 Acknowledgement

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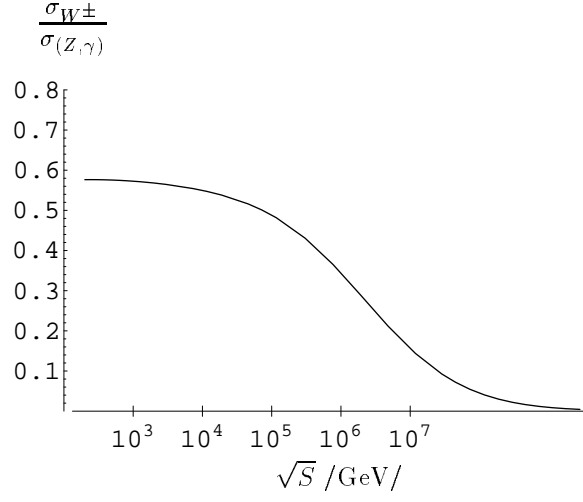


Figure 3: *Total energy dependence of W^\pm to (Z, γ) rate in e^+e^- annihilation.*

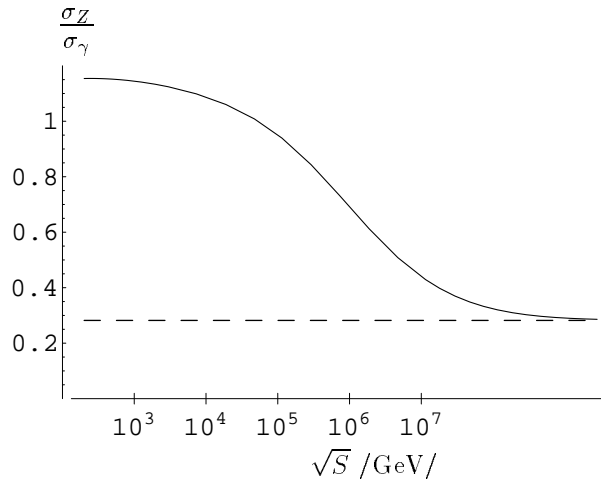


Figure 4: *Total energy dependence of Z to γ rate in e^+e^- annihilation. The dashed line shows the asymptotical value of the ratio: $\tan^2 \theta_W \approx 0.28$.*